SWAMI VIVEKANAND ACADEMY

Class – XII – Maths –Test Paper – Date: 19/12/2019

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. The system of equations, $x + 2y = 5$,$4x + 8y = 20$ has

   a. no solution

   b. none of these

   c. a unique solution

   d. infinitely many solutions

2. If A and B are invertible matrices of order 3, then det(adj A) =

   a. $|A|^2$

b. None of these

c. (det A)^2

d. 1

3. $\underset{x\to\pi}{Lt}\ \frac{1+\cos^3 x}{(x-\pi)^2}$ is equal to

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{3}{2}$

d. None of these

4. Differential coefficient of a function f(g(x)) w.r.t. the function g (x) is

a. f'(g (x))

b. None of these

c. $\frac{f'(g(x))}{g'(x)}$

d. f '(g (x)) g' (x)

5. General solution of $\frac{dy}{dx}+y=1\ (y\neq 1)$ is

a. $y = B + Ae^{-x}$

b. $y = 1 + Ae^{-x}$

c. $y = 1 + Ae^{-3x}$

d. $y = 1 + Ae^{x}$

6. $\cos\left(\cos^{-1}\left(\frac{7}{25}\right)\right) =$

a. $\frac{25}{7}$

b. None of these

c. $\frac{25}{24}$

d. $\frac{24}{25}$

7. If A and B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, Find P(not A and not B).

a. $\frac{1}{8}$

b. $\frac{2}{5}$

c. $\frac{3}{5}$

d. $\frac{3}{8}$

8. $\int_{0}^{\pi/2} \log(\tan x)\, dx$ is equal to

a. $\int_{0}^{\pi/2} \log(\cot x)\, dx$

b. 1

c. $-\frac{\pi}{2}\log 2$

d. $\frac{\pi}{2}\log 2$

9. Find the coordinates of the foot of the perpendicular drawn from the origin to 3y + 4z – 6 = 0

a. $\left(1, \frac{18}{25}, \frac{24}{25}\right)$

b. $\left(0, \frac{18}{25}, \frac{27}{25}\right)$

c. $\left(0, \frac{18}{25}, \frac{24}{25}\right)$

d. $\left(0, \frac{19}{25}, \frac{24}{25}\right)$

10. If θ is the angle between two vectors $\vec{a}\ and\ \vec{b}$, then $\vec{a}.\vec{b} \geqslant 0$ only when

   a. $0 < \theta < \frac{\pi}{2}$

   b. $0 \leqslant \theta \leqslant \pi$

   c. $0 < \theta < \pi$

   d. $0 \leqslant \theta \leqslant \frac{\pi}{2}$

11. Fill in the blanks:

   A function f : X $\rightarrow$ Y is said to be an _______ function, if every element of Y is image of some element of set X under f.

12. Fill in the blanks:

   The probabiltiy of drawing two clubs from a well shuffled pack of 52 cards is _______.

13. Fill in the blanks:

   If A is symmetric matrix, then B'AB is _______ matrix.

14. Fill in the blanks:

   The function A(x) denotes the _______ function and is given by A (x) = $\int\limits_a^x f(x)dx$.

OR

   Fill in the blanks:

   The value of integral is $\int_0^1 \frac{dx}{e^x+e^{-x}}$ is _______.

15. Fill in the blanks:

   Any point in the feasible region that gives the optimal value of the objective function is called an _______ solution.

OR

Fill in the blanks:

The process of obtaining the optimal solution of the linear programming problem is called ______.

16. Evaluate $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

17. If a line has the direction ratios -18, 12, -4 then what are its direction cosines

18. Evaluate $\int_0^{\pi/4} \tan x dx$.

OR

$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

19. Find the length of the longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing.

20. Find the magnitude of the vector $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$.

Section B

21. Find the value of parameter α for which the function $f(x) = 1 + \alpha x, \alpha \neq 0$ is the inverse of itself.

22. Find $\frac{dy}{dx}$, if $x^3 + x^2 y + xy^2 + y^3 = 81$

OR

Find the second order derivative of the function $e^x \sin 5x$

23. Find sine of the angle between the vectors. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

24. Find point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents is parallel to x-axis.

OR

Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$

25. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

26. A Box of oranges is inspected by examining three randomly selected oranges drown without replacement. If all the three oranges are good, the box is approved for sale, other wise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approve for sale?

Section C

27. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

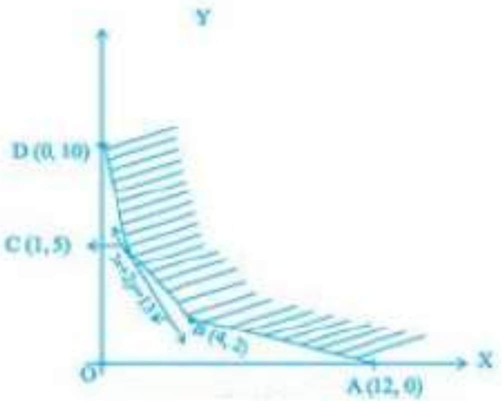
28. Find the relationship between a and b, so that the function f defined by f(x)= $\begin{cases} ax+1, \text{ if } x \leq 3 \\ bx+3, \text{ if } x > 3 \end{cases}$ is continuous at x = 3.

OR

Prove that the greatest integer function [x] is continuous at all points except at integer points.

29. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of a number of defective bulbs?

30. Determine the minimum value of Z = 3x + 2y (if any), if the feasible region for an LPP is shown in Fig.



31. If y = sin(sin x), prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$.

OR

Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation: y - cos y = x : y sin y + cos y + xy' = y

32. Evaluate $\int_0^1 \frac{\log|1+x|}{1+x^2} dx$.

Section D

33. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

OR

Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that $A_2 - 4A + 7I = 0$. Using this result calculate A^5 also.

34. An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

35. Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

OR

Find the area bounded by the lines y = 4x + 5, y = 5 - x and 4y = x + 5.

36. Find the coordinate where the line thorough (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7.